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ABSTRACT

This publication was developed as a portion of a two-semester sequence commencing at either the sixth or seventh term of the undergraduate program in electrical engineering at the University of Pittsburgh. The materials of the two courses, produced by a National Science Foundation grant, are concerned with power conversion systems comprising power electronic devices, electromechanical energy converters, and associated logic configurations necessary to cause the system to behave in a prescribed fashion. The emphasis in this portion of the two course sequence (Part 1) is on electric machinery analysis. This publication is the problem manual for Part 1, which provides problems included in the first course. (HM)

PROBLEM MANUAL

for

POWER PROCESSING, PART 1

"Electric Machinery Analysis"

Howard B. Hamilton

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1970

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CHAPTER 1 - BASIC PRINCIPLES

1-1. An ammeter of the moving-iron type is shown in Figure 1-1. The curved soft iron piece is drawn into the stationary coil. This motion is opposed by the torsional spring. The inductance of the coil is $L = 8 + 25\theta$ micro henries where θ is deflection in radians.

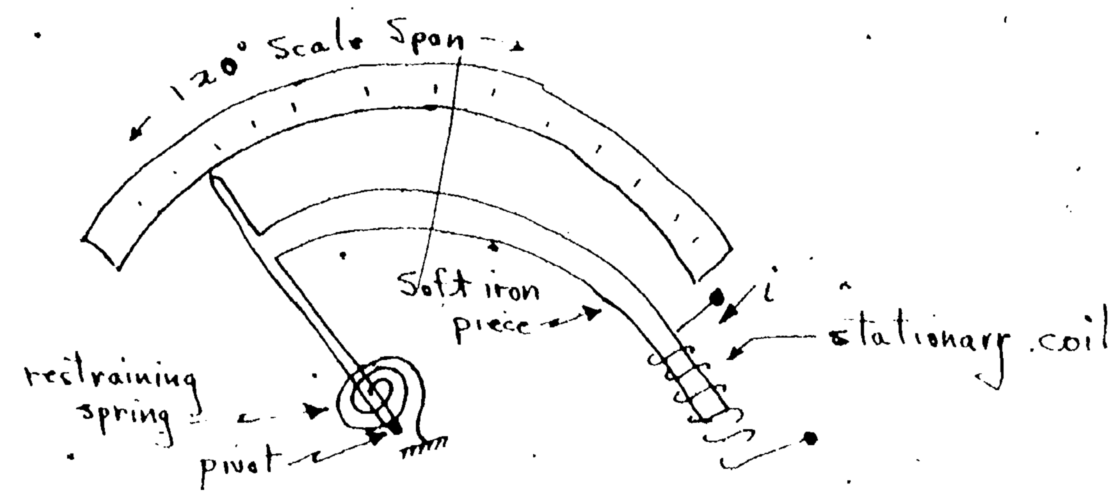


Figure 1-1

a) Show that the instrument measures the root mean square value of the current.
b) Find the spring constant, K, for a 120° deflection of the pointer when 5 amperes rms current flows.
c) If coil resistance is 0.01 ohms, what will be the voltage drop across the ammeter when 5 amperes at 60 hz flows through the ammeter?

1-2.

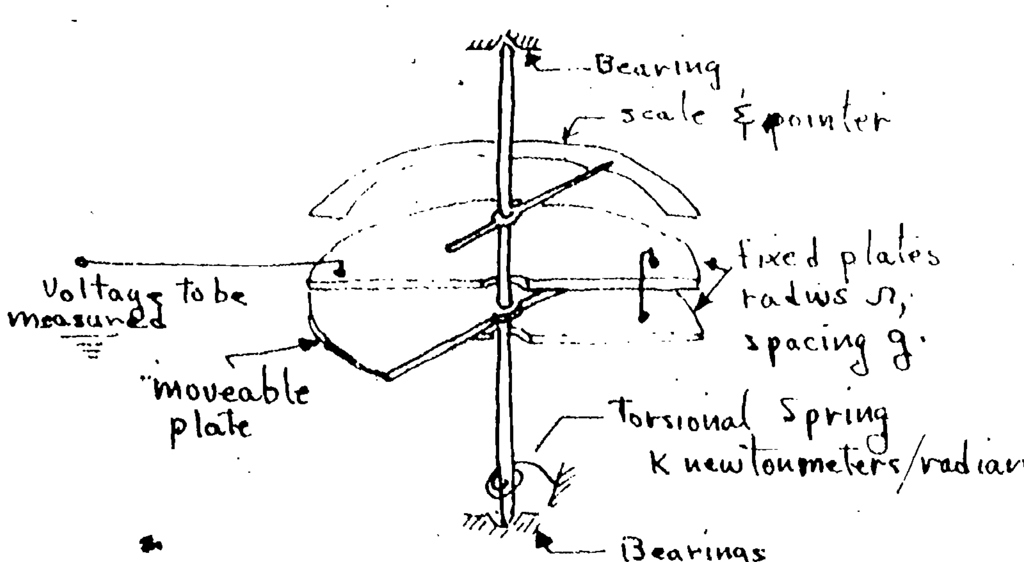


Figure 1-2

Figure 1-2 depicts the movement of an electrostatic voltmeter used to measure relatively high voltage from high impedance sources. Application of voltage causes the moveable plate to enter the spacing between the fixed plates. The position of the moveable plate with no voltage applied is where it is just about to enter the space between the fixed plates.

a) For a maximum d.c. voltage of V_d and π radians of pointer rotation, find K, the spring constant in terms of the various parameters.
b) What will be the deflection for any voltage V?
c) Assume a sinusoidal voltage $V_m \sin \omega t$ is impressed. What will the deflection be?
d) When a d.c. voltage of V_d is impressed, the angular deflection is given by:

$$\theta = \pi(1 - \epsilon^{-t/T})$$

Calculate the energy extracted from the source, the energy stored in the spring and the energy stored in the capacitance. Is energy conserved here?

1-3. The rotor and stator of a simplified 4 pole machine are shown in Figure 1-3.

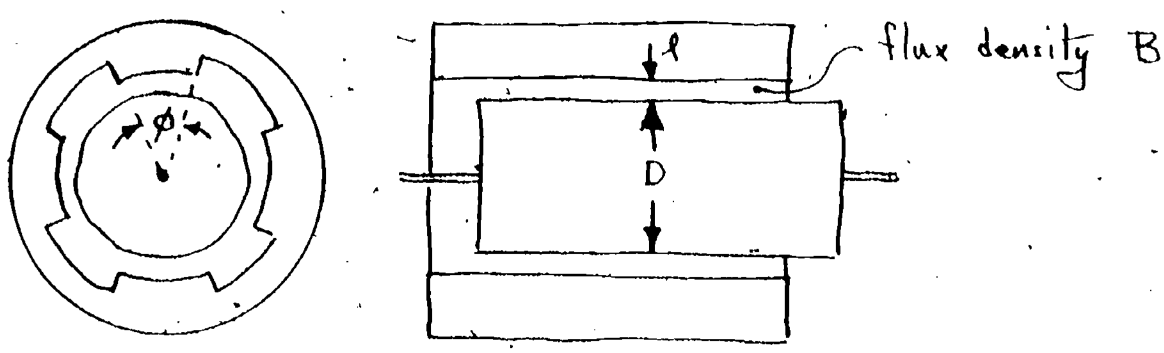


Figure 1-3

Manufacturing tolerances result in an axial displacement of the rotor as shown above. Machine dimensions are:

ℓ = 0.025 meters　　B = 0.85 webers/meter^2
D = 0.250 meters　　ϕ = 1 radian

If the slots are neglected, the air gap length ℓ can be considered constant. Neglect fringing of the flux. Find the axial force tending to correct the misalignment.

1-4. A cylindrical magnet structure is shown in Figure 1-4. The moveable plunger has a cross sectional area of 25×10^{-4} meters^2. It slides horizontally through a circular opening in the magnetic housing. The air gap between plunger and housing is negligible.

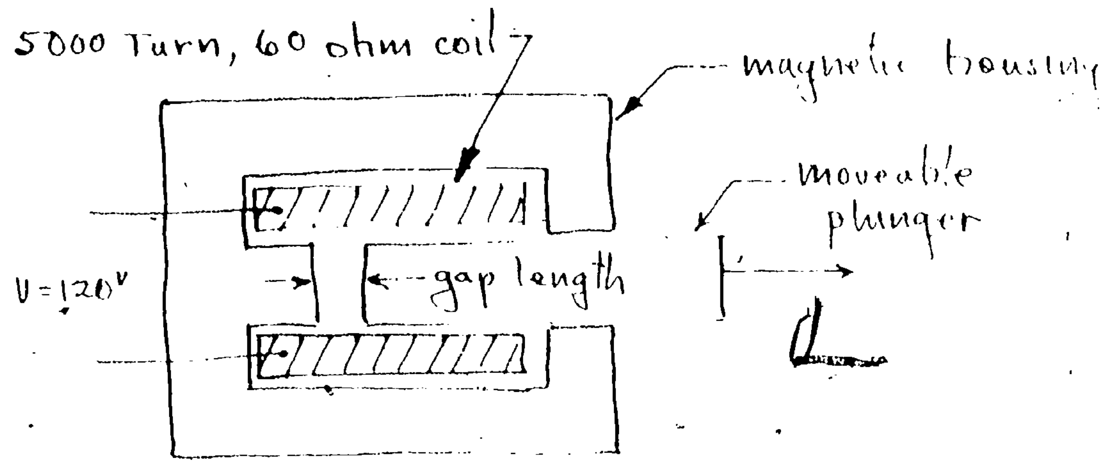


Figure 1-4

The magnetic material is of extremely high permeability and saturates abruptly at 1.5 webers/meter^2.

a) Determine an analytical expression for the static force on the plunger as a function of gap length, noting that a saturation flux density will occur. Plot force vs gap length.
b) If the plunger moves slowly from a gap length of 1.5 cm to the fully closed position what mechanical work is done?
c) If the plunger moves so quickly from an initial gap of 1.5 cm that the flux linkages of the coil remain constant during motion how much mechanical work is done?

1-5. An elementary reluctance motor is shown in Figure 1-5.

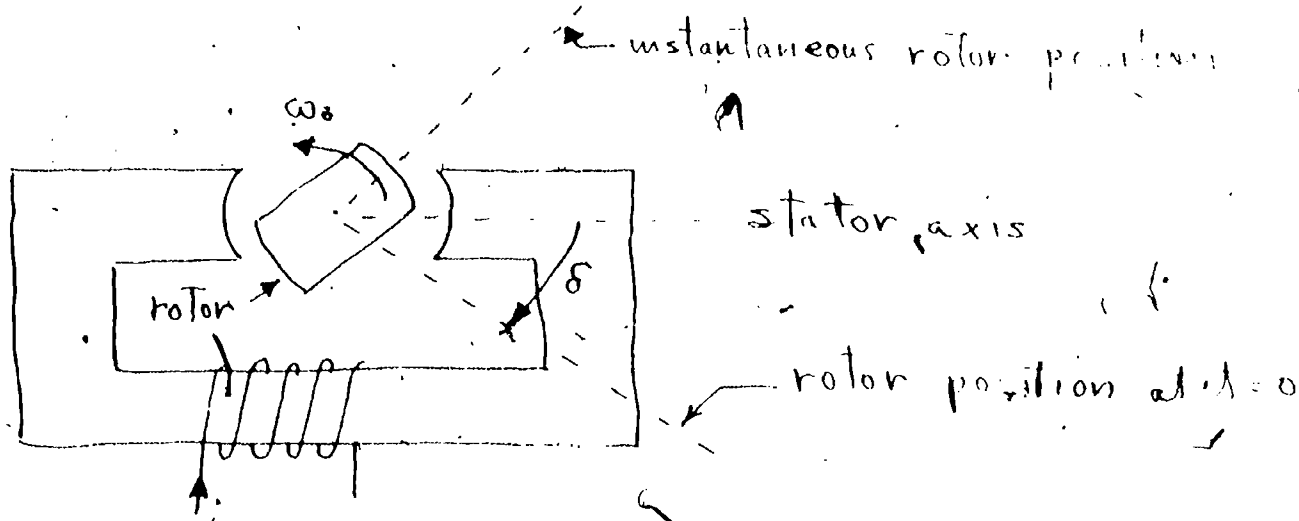


Figure 1-5

The sinusoidal current, i, of angular velocity ω causes an instantaneous flux, ϕ, as:

$$\phi = \phi_m \cos \omega t$$

The instantaneous reluctance $\mathcal{R}$ is a function of rotor position, θ_o. If a sinusoidal variation of $\mathcal{R}$ is assumed

$$= \frac{1}{2}(\mathcal{R}_q + \mathcal{R}_d) - \frac{1}{2}(\mathcal{R}_q - \mathcal{R}_d) \cos 2\theta_o$$

where the d and q subscripts denote rotor position along the d axis, or stator axis and in quadrature with the stator axis.

a) Show that for net average torque to be produced the rotor angular velocity must equal the electrical angular velocity, i.e. $\omega = \omega_o$.

b) Show that T_{ave}, the average value of torque, is given by

$$T_{ave} = \frac{1}{8} \phi_m^2 (\mathcal{R}_q - \mathcal{R}_d) \sin 2\delta$$

c) For a winding of negligible resistance and energized from a sinusoidal voltage source of rms value V, it can be shown that:

$$V = 4.44 \, f \, N \phi_m$$

Also, from the definition of inductance, L:

$$L = \frac{N\phi}{I} = \frac{N^2\phi}{NI} = \frac{N^2}{\mathcal{R}}$$

where f is electrical frequency and N is the number of turns comprising the winding.

For a reluctance motor with stator winding N = 2000 turns, L_d = 1.25 h, L_q = 0.5 h energized from a 60 hz, 120 volt rms source, calculate the torque developed.

1-6. Figure 1-6 depicts, in cross section, a machine with rotor wind FF' and two identical stator windings aa' and bb' which are in quadrature.

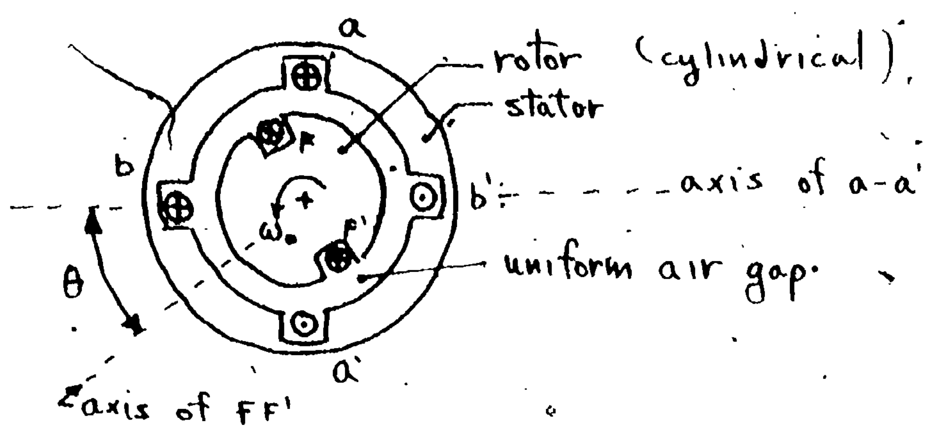


Figure 1-6.

The self inductances of the stator windings are $L_a = L_b$ and the self inductance of the rotor winding is L_f henrys. The mutual inductances L_{af} and L_{bf} are:

$$L_{af} = L \cos \theta \qquad L_{bf} = L \sin \theta$$

The resistance of each stator winding is r ohms.

a) Find a general expression for instantaneous torque as a function of i_a, i_b, i_f, θ and the inductances.
b) If the rotor is stationary at $\theta = 0$ and constant currents of value $i_a = i_b = i_f$ are applied, what is the initial torque? What is the final angle of the rotor?
c) What is the maximum torque that can be developed for constant currents $i_a = i_b = i_f$? At what angle does this occur?
d) Assume the rotor is excited by constant current I_f and the stator windings are excited by sinusoidal currents with a time separation of $\pi/2$ radians. Thus;

$$i_a = \sqrt{2}\, I \cos \omega t$$

$$i_b = \sqrt{2}\, I \sin \omega t$$

The angle θ is given by:

$$\theta = \omega_o t - \delta$$

where δ is the position of the rotor at $t = 0$. We now have an elementary cylindrical rotor polyphase (2) motor. What relationship between ω and ω_o must exist if average torque is produced?

e) What is the value of the average torque under the conditions of d).
f) Find an expression for the instantaneous terminal voltage applied to windings aa' and bb'.

1-7. Figure 1-7 depicts a two phase polyphase synchronous machine with salient rotor, rather than a cylindrical rotor as in problem 1-6.

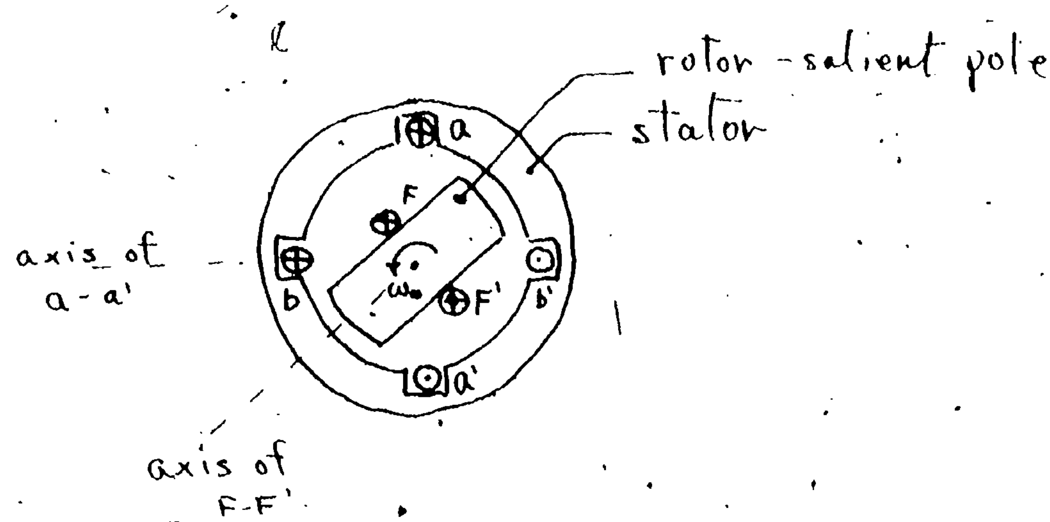


Figure 1-7

Because of the nonuniform air gap the self and mutual inductances of the stator windings are functions of the rotor position as is the mutual inductances of the rotor. The self inductance of the rotor is constant. The inductances are:

$$L_a = L_o + L \cos 2\theta$$

$$L_b = L_o - L \cos 2\theta$$

$$L_{ab} = L \sin 2\theta$$

$$L_{af} = M \cos \theta$$

$$L_{bf} = M \sin \theta$$

$$L_f = L_f$$

The rotor is excited with a direct current I_f. The currents in the stator windings are:

$$i_a = \sqrt{2}\, I \cos \omega t \qquad i_b = \sqrt{2}\, I \sin \omega t$$

The rotor is revolving at speed $\omega_o = \omega$ and rotor position is:

$$\theta = \omega_o t - \delta$$

where δ is rotor position at $t = 0$. Show that the average torque is a constant (not a function of time) and is composed of a component proportional to $\sin \delta$ and a component proportional to $\sin 2\delta$. The latter is the "reluctance" torque and the former is the "synchronous" torque.

2-1. A d.c. machine is tested by driving at 1200 rpm and exciting the field winding with 1.5 amperes. The "open circuit" armature voltage is 200 volts. The armature resistance is measured as 0.10 ohms. A resistance load of 2.5 ohms is connected across the armature circuit and the field current is adjusted to cause a current of 100 amperes to flow in the 2.5 ohm load. Speed is constant at 1200 rpm.

a) What is the value of field current necessary?
b) What is the electrical power output of the machine?
c) What is the mechanical power input to the machine?

2-2. The armature of the machine from 2-1 is connected across a 250 volt d.c. source to run as a motor. The field is excited with 1.5 amperes. A torque load of 15 newton meters is connected to the shaft. The speed will adjust iseflf to permit the flow of the necessary magnitude of armature current which in turn results in the proper value of shaft torque.

a) What is the armature current?
b) What is the resulting speed?
c) If the field excitation is increased to 2.0 amperes and the torque is maintained constant what are the new values of steady state current and speed?
d) If the torque is reduced to 10 newton meters and the excitation maintained at 2.0 amperes what are the values of speed and armature current?
e) If the torque is 10 newton meters, the field excitation at 2.0 amperes, and the applied source voltage is reduced to 125 volts d.c, what are the values of speed and armature current?

2-3. a) Compare the effect on the speed of a d.c. motor of varying the line voltage with that of varying only the armature terminal voltage, so that the field current remains fixed.

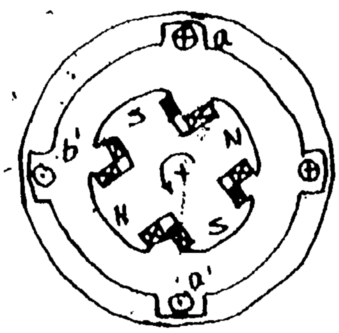
b) Compare both these effects with that of varying only the field current, the armature terminal voltage remainng fixed.

2-4. State how the armature current and speed of a d.c. motor would be affected by each of the following changes in the operating conditions: (In each case, only brief quantitative statements of the order of magnitude of the changes are desired, e.g., "speed approximately halved").

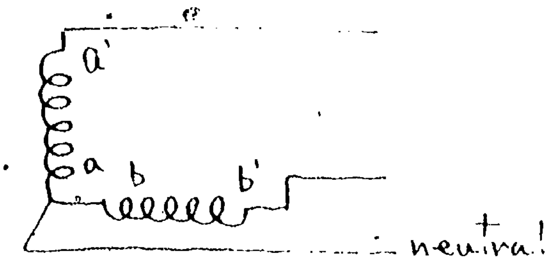
a) Halving the armature terminal voltage, the field current and load torque remaining constant.
b) Halving the armature terminal voltage, the field current and horsepower output remaining constant.

c) Doubling the field flux, the armature terminal voltage and load torque remaining constant.
d) Halving both the field flux and armature terminal voltage, the horsepower output remaining constant.
e). Halving the armature terminal voltage, the field flux remaining constant and the load torque varying as the square of the speed.

2-4. A small elementary generator is depicted in Figure 2-5.



(a)



(b)

Figure 2-5

Coils aa' and bb' represent concentrated coils of 10 turns each. All the turns in each coil are connected in series and the coils connected as shown in (b) to form a 2 phase system. The flux resulting from each pole is sinusoidally distributed in space and is 50 megalines per pole. The rotor is driven at 1800 rpm.

a) Find the rms generated voltage to neutral.
b) Determine the line-line rms voltage.
c) What is the frequency of the generated voltage?
d) Take zero time as the instant when the positive flux linkages (from a N pole) with aa' are a maximum. Write a consistent set of time equations for the two phase voltages aa' and bb'.
e) Write a consistent set of time equations for the voltages a'-b' and b'-a'.

2-6. A motor has its rotor supplied with direct current and its stator (armature) with alternating current from a constant voltage source. The load on the motor is a constant torque load. Describe the effect of increasing the field current on the following:

a) The magnitude of the resulting flux wave.
b) The magnitude of the armature current.
c) The magnitude of the armature current in phase with the armature voltage.
d) The space angle between armature mmf and resultant flux wave.

3-1. A d.c. machine rated 250 volts, 1750 rpm, 4 pole has 2 parallel paths through the armature. The armature has 35 slots with 6 coil sides per slot and 2 turns per coil. The average length of one turn is 2.5 feet and the conductor has a resistance of 1.0 ohm per 1000 feet.

a) Find the flux per pole to produce 250 volts at rated speed.
b) Determine the resistance of the armature winding.
c) The machine is to be operated as a generator supplying 50 amperes into a load. If the voltage drop is 2.0 volts across the carbon brushes and the terminal voltage, i.e., voltage across the load, is 250 volts what value of flux per pole is required? (Speed at 1750 rpm).
d) If the flux per pole was at the value in a) what speed would be necessary to supply 50 amperes at 250 volts terminal voltage?
e) If the machine was running as a motor with 250 volts applied and the load such that 50 amperes of armature current resulted, what value of flux per pole would yield a speed of 1500 rpm?
f) What is the torque developed under the conditions of e)?

3-2. The full load torque angle of a synchronous motor at rated voltage and frequency is $\pi/6$ electrical radians. Neglect the effects of armature resistance and leakage reactance and consider that field excitation remains constant. How will the torque angle be affected by the following?

a) Frequency reduced 15%, load torque constant.
b) Frequency reduced 15%, load power constant.
c) Applied voltage reduced 10%.
d) Frequency and applied voltage reduced 15% with load torque constant.
e) Frequency and applied voltage reduced 15% with load power constant.

4-1. A 100 kw, 250 volt d.c. generator has an armature resistance of 0.025 ohms. Armature inductance is 0.01 henries. Brush drop is negligible. Field resistance is 25 ohms, field inductance is 2 henries. The generator is driven at a constant speed of 1200 rpm. On open circuit, a shunt field current of 5.0 amperes results in rated voltage at the terminals. The shunt field has 1000 turns.

a) For the steady state condition, find an expression for field current as a function of load current for constant terminal voltage of 250 volts as load varies from 0 to 150% of rated.

b) A load of R = 0.625 ohms resistance and 0.1 henrys inductance is switched into the armature circuit at t = 0. Find the terminal voltage as a function of time if the speed and field excitation remains constant.

4-2. Consider the generator of problem 4-1 to be connected as a self excited generator with 25 ohms of external resistance connected in series with the field winding. With the field circuit open and the generator driven at rated speed the terminal voltage is 10 volts. This is due to residual magnetism. At t = 0, the field circuit is connected across the armature terminals. Formulate an algorithm for a digital computer which will find terminal voltage as a function of time.

4-3. An additional field circuit to be connected in series with the armature is to be added to the machine in problem 4-1. This winding has 4 turns and a resistance of 0.001 ohms. What resistance should be placed in parallel with this "series" field in order to provide the amount of current through the series field which will result in additional excitation such that the terminal voltage is 250 volts at both no load and at rated load? The machine is then "flat compounded". What is the terminal voltage at 50% load? At 150% load?

4-4. An adjustable speed d.c. shunt motor for a rolling mill has the following constants and ratings:

1000 hp at 750 rpm, 1400 amperes
300 hp at 225 rpm, 420 amperes
600 volts
armature resistance = 0.02 ohm
armature inductance = 0.0008 h
Wr^2 = 30,000 lb-ft^2 (motor and load)

It is desired to study the speed transient after sudden application of full load torque. Assume that the motor is initially unloaded, i.e., $i_a = 0$, and the torque applied to be independent of speed. Find $\omega(t)$ for operation at both maximum and minimum speed. What is the difference in the character of the response at the two speeds?

4-5. A d.c. shunt motor with constant field excitation has its speed controlled by a single step of resistance, R, connected in series with the armature. Load torque is of the form $T_L = k\omega$ and armature inductance must be considered. At $t = 0$, R is short circuited so that speed increases. Find $\omega(t)$ for the general case and sketch the form of the solution if the system is overdamped.

4-6. A d.c. motor with constant excitation, an inertia load only, and negligible armature inductance is initially at rest. At $t = 0$, a step voltage is impressed on the armature. Find the energy dissipated in the armature circuit as heat during the period the motor accelerates to steady state speed. Express your answer in terms of stored energy in the rotor. Note that at steady state speed, $\omega = V/K$. Assume the motor is running in one direction with rated voltage applied. The motor is reversed in direction by applying opposite polarity armature voltage to the rotor. Calculate energy expended in the armature resistance under this condition. This is referred to as "plugging". Compare the two values of energy expended and draw a conclusion about the severity of reversing duty.

4-7. A d.c. series motor operates at 750 rpm with a line current of 80 amperes from a 230 volt source. Total armature circuit resistance is 0.25 ohms. This includes the series field resistance. Because of nonlinearity of the magnetic circuit, the flux at a current of 20 amperes is 40% of that resulting from a current of 80 amperes. What is the speed at 20 amperes, 230 volts?

4-8. A d.c. series motor drives a constant torque load (torque required is independent of speed). Neglect armature circuit resistance and saturation effects.

a) If the applied voltage is decreased from 100% to 75% of rated, what is the percentage of change in speed?
b) Repeat a) except for a d.c. shunt motor rather than the series type.

4-9. An automatic starter is to be designed for a 15 hp, 230 volt shunt motor whose armature circuit resistance is 0.16 ohms. The field circuit resistance is 115 ohms. When delivering rated hp output armature current is 56 amperes. When loaded until armature current is 32 amperes, motor speed is 1100 rpm.

The motor is connected to a load which requires torque proportional to speed and takes 15 hp at rated speed. The starter is to connect the field circuit across 230 volts but to insert sufficient resistance in series to limit the armature current to 125 amperes and when it falls to 56 amperes some resistance is to be cut out to maintain the current always between 125 and 56 amperes during the starting period. Determine the number of steps of resistance required and the resistance of each of the steps.

4-10. A d.c. shunt motor driving a constant torque load at steady state speed ω_s is to be stopped by "dynamic braking". This is accomplished by instantaneously switching the armature terminals from the source to a braking resistor with field excitation maintained constant. The scheme is shown in Figure 4-10.

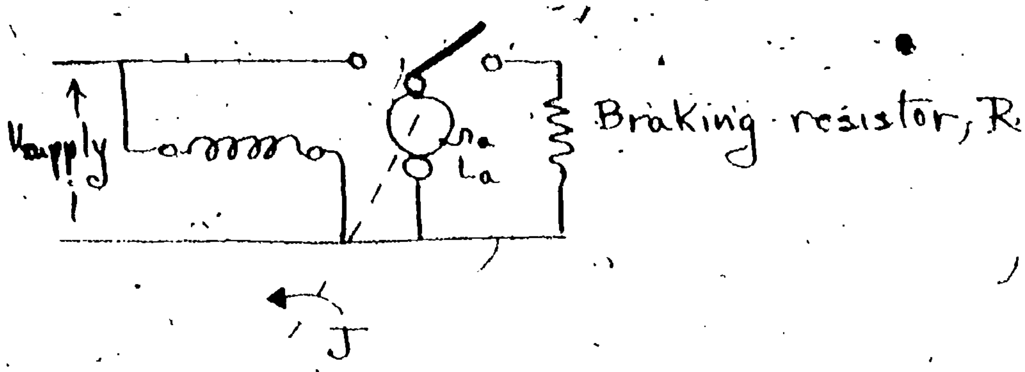


Figure 4-10

Find an expression for $\omega(t)$ during braking. Neglect all damping other than that due to power expended in r_a, R.

4-11. Figure 4-11 illustrates a "shunted armature" connection used to obtain a specific speed torque characteristic in a d.c. shunt motor. Find an expression for the steady state speed torque relationship if armature resistance is negligible. Sketch the relationship indicating limiting values of speed and torque.

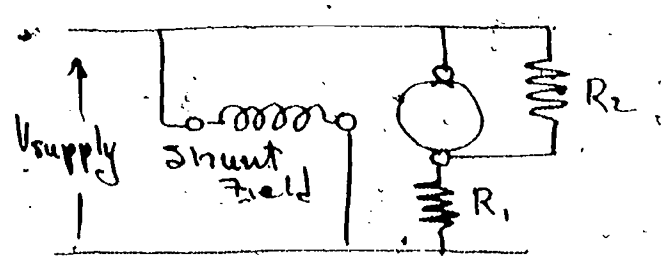


Figure 4-11

4-12. Figure 4-12 illustrates a shunted armature connection for use with a series motor. Repeat 4-11 for the series motor.

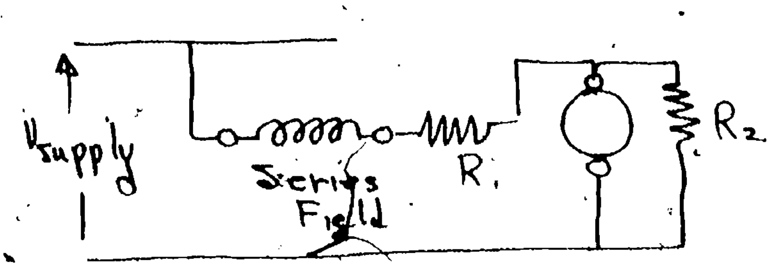


Figure 4-12

4-13. A d.c. series motor has the following rating and parameters:

150 hp, 600 volts, 600 rpm, 206 amperes
brush voltage drop = 2 volts
armature resistance = 0.04 ohms
armature inductance = 0.03 henry
series field resistance = 0.105 ohms
series field inductance = 0.08 henry

It is tested by driving it with another motor at 400 rpm and passing 216 amperes (from an external source) through the series field. The resulting open circuit voltage is measured as 400 volts.

a) What is the full load efficiency?
b) How does this subdivide into copper loss (electrical) and rotating loss (mechanical)?
c) Find the steady state speed torque characteristic.
d) Repeat (c) if the series field is shunted by a 0.105 ohm resistor.

4-14. By using a computer, determine $\omega(t)$ if rated torque is connected and the motor from 4-13 is energized with 150 volts for 15 seconds and then full voltage (600 volts) is applied. Total $Wk^2 = 45000$ lb-ft^2.

4-15. Bids are requested for a d.c. motor for a pump drive. The pump specifications call for a drive speed of 850 rpm, 820 gal of water per minute at 85 feet head. Pump efficiency is to be 70%.

a) Based on 1 gal of water weighing 8.32 lb, determine the size of the motor to be bid.

Two bids are received. The first is for $600 for the motor which has an efficiency of 90%. The second is for a motor priced at $500 but with an efficiency of 88%. The pump will operate 6 days/week, 14 hours/day. Energy cost is $0.03 kw hr.

b) Evaluate the two bids and justify the purchase of one or the other.

4-16. A 1,700-rpm centrifugal pump, driven by a 50-hp shunt motor, discharges 200 ft^3 of water a minute against a head of 90 ft. Under these conditions, the following data are taken:

Motor terminal voltage = 228 volts
Motor field current = 3.25 amperes
Motor armature current = 195 amperes
Motor speed = 1,700 rpm

When the pump is disconnected from the motor, the following data are taken:

Motor terminal voltage = 230 volts
Motor armature current = 10 amperes
Motor field current = 3.57 amp
Motor speed = 1,700 rpm

Motor-armature circuit resistance at operating temperature exclusive of brushes is 0.100 ohm. Voltage drop at the brushes can be taken as 2 volts.

What is the efficiency of the pump?

4-17. In continuous rolling mills, the rolls through which the bar passes during rolling are in tandem with each stand, or roll, driven by a motor. D.C. Motors are used because of the ease with which they can be controlled. The transient changes of the motor speed under suddenly applied loads as the bar enters one stand after another may seriously affect the quality of the product. In particular, the impact speed drop which occurs at the maximum of the transient oscillation is of major importance. For analysis purposes, consider a single motor supplied by a d.c. source. The motor is separately excited and compensated so that armature reaction effects are negligible. With the motor running without external load and the system in the steady state, a bar enters the stand at $t = 0$, causing the load torque to be increased suddenly from zero to T. The following numerical values apply:

Source voltage = 390 volts
Armature circuit inductance, $L = 0.0077$ henry
Armature circuit resistance $R = 0.035$ ohm
Moment of inertia of motor armature and connected rolls, all referred to motor speed, $J = 42.2$ kg-m^2
Electromechanical conversion constant for motor, $K_m = 4.23$ newton-m/amp
No-load armature current, $i_o = 35$ amp
Suddenly applied torque, $T = 2,040$ newton-m

Determine the following quantities:

a) The undamped angular frequency of the transient speed oscillations.
b) The damping ratio of the system.
c) The time constant of the system, in seconds.
d) The initial speed, in rpm.
e) The initial acceleration, in rpm per second.
f) The ultimate speed drop, in rpm.
g) The impact speed drop, in rpm.

5-1. A 1 Kw metadyne has the following constants

control field resistance $r_f = 25$ ohms
control field inductance $L_f = 5$ henry
$\omega L_{df} = K_d = 240$
armature circuit resistances $r_q = r_d = 4$ ohms
$\omega L_q = K_q = 120$
neglect armature circuit time constants

This unit supplies a 50 ohm resistance load at a voltage of 150 volts.

a) What is the power input to the control field.
b) What is the amplification.

5-2. Repeat 5-1 for an amplidyne configuration, i.e., a compensating winding is utilized to counteract the demagnetizing effect of the d axis current.

5-3. The metadyne of 5-1 has a constant voltage, applied to the field circuit and the load resistance is varied from 0 to 100 ohms. Plot the ratio of load voltage to field voltage as a function of load resistance. Plot the output power as a function of load resistance.

5-4. The metadyne of 5-1 supplies power to a 40 ohm load and is in the steady state. The field voltage is 25 volts. At t = 0 the voltage is increased by a step function increase up to 50 volts. Find the load current as a function of time.

5-5. The amplidyne configuration of the metadyne is often used to provide excitation for conventional ac or dc generators. Rather simple circuitry can be incorporated to include automatic voltage regulation. Such a system is shown in Figure 5-1.

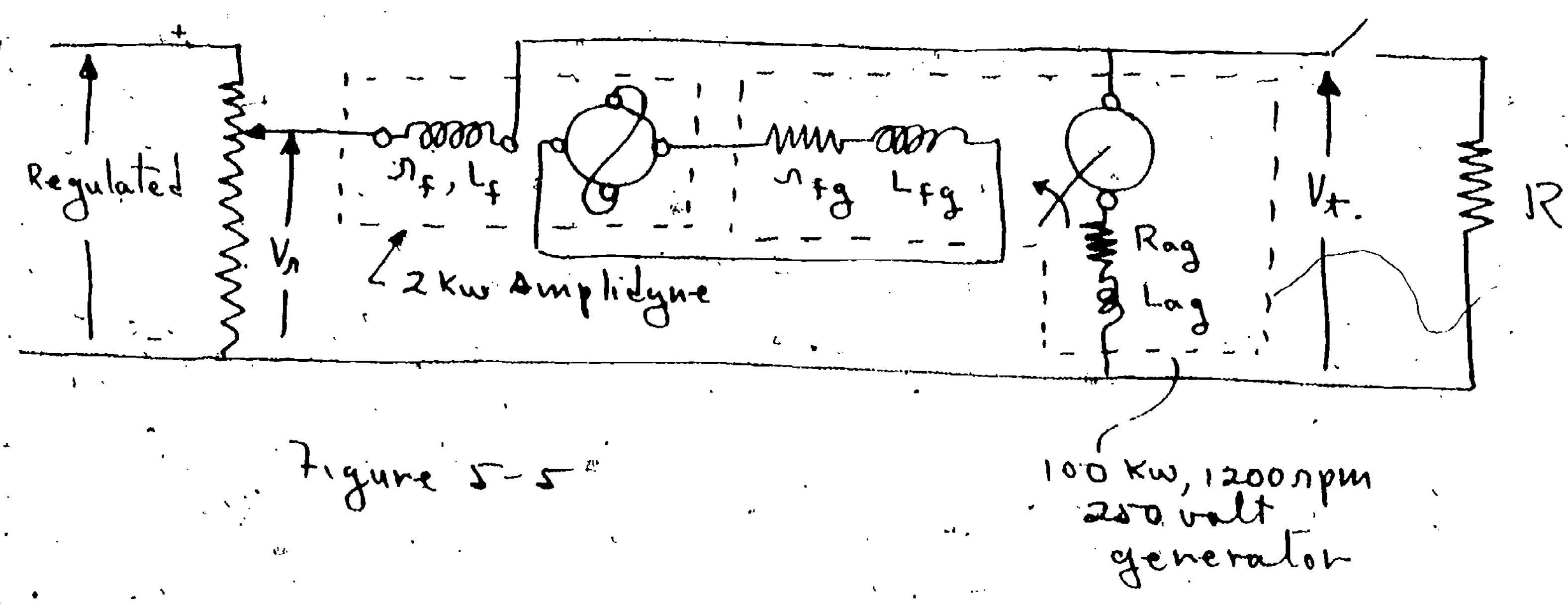


Figure 5-5

The machine constants are as follows:

Generator:

V_t rated = 250 volts
R_{ag} = 0.03125 ohms
L_{ag} = negligible
r_{fg} = 20 ohms
L_{fg} = 1 henry

K_g = voltage constant = 25 volts/amp.

Amplidyne:

r_f = 20 ohms
L_f = 2 henry
r_d = 4 ohms
r_q = 4 ohms

K_d = 30 volts/amp.
K_q = 60 volts/amp.

The quadrature axis armature time constant is negligible and $L_d << L_{fg}$.

a) Find the value of V_r to yield a steady state value of terminal voltage, V_t = 250 volts when the generator is delivering rated output.
b) For the value of V_r in a), what is the no load terminal voltage?
c) What is the voltage regulation with and without the regulator circuitry?
d) What is the damped angular velocity of the oscillations in V_t following a disturbance?
e) What is the damping ratio of the response?
f) What is the time constant of the response?

5-6. In the derivation of the equations for cross field machines it was assumed that the time constants associated with the D and Q axis were negligible. Derive the transfer function $V_a(s)/V_f(s)$ if these time constants are not negligible. Construct the block diagram for this condition.

5-7. Assume an amplidyne with the following parameters.

$T_d = 0.05$
$T_q = 0.333$
$T_f = 0.2$

$r_d = 4$

Using Routh's criteria, determine the load condition which results in an unstable terminal voltage.

CHAPTER VI - SYNCHRONOUS MACHINES

6-1. Derive the equivalent of (VI-16) for a two phase machine which has 2 distributed windings with magnetic axis displaced $\pi/2$ radians and phase currents in quadrature.

6-2. Draw, to scale, the stator mmf distributions for a two pole, two phase machine at the following instants of time:

a) When the current in phase a is zero
b) When the current in phase b is zero
c) An arbitrarily chosen instant not corresponding to a) or b).

6-3. Derive the equivalent of (VI-16) for a q phase machine.

6-4. This problem is concerned with analysis of a 2-phase synchronous machine instead of the 3-phase machine of the text. Consider that the machine is idealized and that there are 2 distributed windings, a and b, on the stator, 1 in each phase, with magnetic axes 90° apart. The salient-pole rotor has only the main-field winding f in the d axis and no winding in the q axis. The angle from the axis of phase a to the d axis is θ. That from the phase b axis to the d axis is $\theta + 270°$ or, what amounts to the same thing, $\theta - 90°$.

a) Write the flux-linkage equations in form corresponding to equations VI-25 through VI-30.
b) Show that the d_q transformation of variables is typified by

$$\begin{bmatrix} i_d \\ i_q \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} i_a \\ i_b \end{bmatrix}$$

Also write the relations for i_a and i_b in terms of i_d, i_q, and θ.
c) Show that the flux linkages are given by

$$\lambda_f = L_f i_f + L_{df} i_d$$

$$\lambda_d = L_{df} i_f + L_d i_d$$

$$\lambda_q = L_q i_q$$

Also identify the d- and q-axis synchronous inductances L_d and L_q in terms of L_{aa0} and L_{aa2}.
d) Show that Equations VI-54, -55 and -51 are correct for the voltages v_f, v_d and v_q in this case.

e) Show that the instantaneous power input to the 2-phase stator is:

$$P8 = v_d i_d + v_q i_q$$

f) Show that motor torque is given by:

$$T = \frac{\text{poles}}{2} (\lambda_d i_q - \lambda_q i_d)$$

6-5. A 2-pole synchronous motor has a 2-phase winding with negligible resistance on the rotor. Its stator has salient poles with a field winding on the d axis having the resistance r_f and excited by a direct voltage V_f. There is a short-circuited stator winding in the q axis.

The motor is running in the steady state with balanced 2-phase voltages given by:

$$v_a = - V_m \sin \omega t \quad \text{and} \quad v_b = - V_m \cos \omega t$$

applied to the rotor windings. The angle from the d axis to the magnetic axis of rotor phase a is:

$$\theta = \omega t + \delta$$

That between the phase b axis and the d axis is 90° greater than this.

a. Show that the d- and q-axis rotor currents into the motor are given by:

$$i_d = \frac{- \omega L_f I_f + V_m \cos \delta}{\omega L_d} \quad \text{and} \quad i_q = \frac{V \sin \delta}{\omega L_q}$$

b. Show that the motor torque is given by

$$T = - \frac{L_f V_f V_m}{\omega L_d r_f} \sin \delta - \frac{V_m^2 (L_d - L_q)}{2 \omega^2 L_d L_q} \sin 2\delta$$

c. Show that the phase a rotor circuit is

$$i_a = i_d \cos \theta + i_q \sin \theta$$

6-6. A 2-pole synchronous machine with a uniform air gap has a Y-connected 3-phase stator winding. The stator windings have a synchronous inductance of 0.01 henry per phase and negligible resistance. The rotor winding has an inductance of 20 henrys and a resistance of 10 ohms. It is connected to a d-c voltage source of 100 volts. The mutual inductance between the rotor and a stator phase when their magnetic axes are aligned is 0.4 henry.

The machine is operating as a generator under steady-state conditions delivering power to a balanced 3-phase system at 1.0 power factor and 400 radians/sec angular frequency. The terminal voltage of each phase is 2300 volts rms. What is the electrical power output of the generator in kilowatts?

6-7. A d-c shunt motor is mechanically coupled to a 3-phase cylindrical-rotor-synchronous generator. The d-c motor is connected to a 230-volt constant-potential d-c supply, and the a-c generator is connected to a 230-volt (line to line) constant-potential constant-frequency 3-phase supply. The 4-pole Y-connected synchronous machine is rated 50 kva, 230 volts, and has a synchronous reactance of 3.0 ohms per phase. The 4-pole d-c machine is rated 50 kw, 230 volts. All losses are to be neglected.

a. If the two machines act as a motor-generator set receiving power from the d-c mains and delivering power to the a-c circuit, what is the excitation voltage of the a-c machine in volts per phase (line to neutral) when it delivers rated kva at 1.00 power factor?

b. Leaving the field current of the a-c machine as in part a, what adjustment can be made to reduce the power transfer (between a-c and d-c) to zero? Under this condition of zero power transfer what is the armature current of the d-c machine? What is the armature current of the a-c machine?

c. Leaving the field current of the a-c machine as in parts a and b, what adjustment can be made to cause 50 kw to be taken from the a-c circuit and delivered to the d-c circuit? Under these conditions, what is the armature current of the d-c machine? What are the magnitude and phase of the current of the a-c machine?

6-8. A synchronous generator is connected to an infinte bus through two parallel 3-phase transmission circuits each having a reactance of 1.2 per unit including step-up and step-down transformers at each end. The synchronous reactance of the generator (which may be handled on a cylindrical-rotor basis) is 1.8 per unit. All resistances are negligible, and reactances are expressed on the generator rating as a base. The infinite-bus voltage is 1.00 per unit.

a. The power output and excitation of the generator are adjusted so that it delivers rated current at 1.0 power factor at its terminals in the steady state. Compute the generator terminal and excitation voltages, the power output, and the reactance power delivered to the infinite bus.

b. The throttle of the prime mover is now adjusted so that there is no power transfer between the generator and the infinite bus. The field current of the generator is adjusted until 0.50-per-unit lagging reactance kva is delivered to the infinite bus. Under these conditions, compute the terminal and excitation voltages of the generator.

c. The system is then returned to the operating conditions of part a. One of the two parallel transmission circuits is disconnected by tripping the circuit breakers at its ends. The generator excitation is kept constant. Will the generator remain in synchronism? After comparing the desired power transfer with the maximum under these conditions, give an opinion regarding the adequacy of the transmission system.

6-9. A 2000 hp, 2,300-volt Y-connected 3-phase 60-cps 20-pole synchronous motor has a synchronous reactance of 8.0 ohms per phase. In this problem cylindrical-rotor theory may be used. All losses may be neglected.

a. This motor is operated from an infinite bus supplying rated voltage at rated frequency, and its field excitation is adjusted so that the power factor is unity when the shaft load is such as to require an input of 1500 kw. If the shaft load is slowly increased, with the field excitation held constant, determine the maximum torque (in pound-feet) that the motor can deliver.

b. Instead of an infinite bus, assume that the power supply is a 2500 kva, 2,300-volt, Y-connected synchronous generator whose synchronous reactance is 10.0 ohns per phase. The frequency is held constant by a governor, and the field excitations of motor and generator are held constant at the values which result in rated terminal voltage when the motor absorbs 1500 kw at unity power factor. If the shaft load on the synchronous motor is slowly increased, determine the maximum torque (in pound-feet). Also determine the armature current, terminal voltage, and power factor at the terminals corresponding to this maximum load.

c. Determine the maximum motor torque if, instead of remaining constant as in part b, the field currents of the generator and motor are slowly increased so as always to maintain rated terminal voltage and unity power factor while the shaft load is increased.

6-10. What per cent of its rated output will a salient-pole synchronous motor deliver without loss of synchronism when the applied voltage is normal and the field excitation is zero, if $x_d = 0.80$ per unit and $x_q = 0.50$ per unit? Compute the per-unit armature current at maximum power.

6-11. A salient-pole synchronous motor has $x_d = 0.80$ and $x_q = 0.50$ per unit. It is running from an infinite bus of $V_t = 1.00$ per unit. Neglect all losses. What is the minimum per-unit excitation for which the machine will stay in synchronism with full-load torque?

6-12. Draw the block diagram for a generator connected to a zero-power-factor lagging load. Consider for these purposes that the load may be represented by a constant reactance x_L. (This may approximate the starting of a large motor, for example).

6-13. A synchronous generator is supplying power to a large system with its field current adjusted so that the armature current lags the terminal voltage. Armature resistance and leakage reactance may be neglected.

The field current is now increased 10 per cent without changing the driving torque of the prime mover. Qualitatively, what changes occur in power output, in magnitude and phase of the armature current, and in magnitude of the torque angle δ ? Explain by means of a phasor diagrams representing the flux and mmf waves.

If, instead of changing the field current, the driving torque of the prime mover is increased 10 per cent, what changes will occur?

6-14. A synchronous motor is operating at half load. An increase in its field excitation causes a decrease in armature current. Before the increase, was the motor delivering or absorbing lagging reactive kva?

6-15. The full-load torque angle δ of a synchronous motor at rated voltage and frequency is 30 electrical degrees. Neglect the effects of armature resistance and leakage reactance. If the field current is constant, how would the torque angle be affected by the following changes in operating conditions?

a. Frequency reduced 10 per cent, load power constant
b. Frequency reduced 10 per cent, load torque constant
c. Both frequency and applied voltage reduced 10 per cent, load power constant
d. Both frequency and applied voltage reduced 10 per cent, load torque constant

6-16. Two small alternators of equal kva rating have per-unit synchronous reactances of 0.6 and 0.8, respectively, on the alternator rating as a base. They are rigidly coupled to the same prime mover and supply power in parallel to a line at rated voltage and frequency. Ignore saturation.

a. The coupling between the machines is such that their no-load terminal voltages are in phase. What is the greatest power that can be delivered to the line without exceeding the current rating of either machine?

b. The coupling is now readjusted so that the machines can jointly deliver a per-unit power of 2.0 without overload. What is the phase angle between their open-circuit terminal voltages?

c. With constant excitation voltages and a governor having no speed-droop, what per-unit current will circulate between the two alternators after dropping the load in (b)? Which machine will act as a motor?

6-17. From the phasor diagram of an overexcited synchronous motor, show that

$$x_q = \frac{V_t \sin\delta - I_a r_a \sin(\phi + \delta)}{I_a \cos(\phi + \delta)}$$

From this relation, the saturated value of x_q can be measured under actual load conditions, by measuring V_t, I_a, power, and δ. The torque angle δ can be measured with a stroboscope.

6-18. From the phasor diagram of an overexcited synchronous motor, show that

$$\tan\delta = \frac{I_a x_q \cos\theta + I_a r_a \sin\theta}{V_t + I_a x_q \sin\theta - I_a r_a \cos\theta}$$

6-19. Derive an expression for the reactive power Q delivered to the infinite bus, in terms of the excitation voltage V_f, the bus voltage V_e, the reactances X_d and X_q, and the angle δ. Neglect resistances, and consider that lagging reactive power delivered to the bus is positive.

6-20. From the phasor diagram of a synchronous machine with constant synchronous reactance x_s operating at constant terminal voltage V_t and constant excitation voltage E_f, show that the locus of the tip of the armature-current phasor is a circle. On a phasor diagram with terminal voltage chosen as the reference phasor indicate the position of the center of this circle and its radius. Express the coordinates of the center and the radius of the circle in terms of V_t, E_f, and x_s.

6-21. Estimating prices for a 200-hp, 1,200-rpm synchronous motor and control are as follows, shaft-driven exciter included:

1.0-power-factor motor, $7,500
0.8-power-factor lead motor, $8,400

Estimated full-load losses are 15 kw for the 1.0-power-factor motor and 16 kw for the other. A proposed application calls for operation at full load for 3,000 hr per year, the motor being shut down the remainder of the time. The incremental power cost is 1.0 cent per kilowatthour, and the total investment charges are 19 per cent per year.

What is the annual cost per reactive kva of the power-factor correction provided by the 0.8 power-factor motor?

6-22. A synchronous motor has a per-unit synchronous reactance (which may be considered constant) of 0.80. It is operating at rated voltage with an excitation voltage 1.3 times the terminal voltage. The armature power input is 0.50 per unit.

a. Determine the power angle, per unit current, and power factor of the motor.

b. For the same power input and terminal voltage, what is another value of excitation which yields the same armature current?

c. Of the excitations in (a) and (b), which gives the higher efficiency of 0.5 per unit power input? Which gives the higher efficiency at 1.0 power input? Which gives the greater margin of stability? Which is more likely to be used in a practical situation? Give reasons for all answers.

6-23. A 3-phase synchronous generator is rated 12,000 kva, 13,800 volts, 0.80 power factor, 60 cps. What should be its kva and voltage rating at 0.80 power factor and 50 cps if the field and armature copper losses are to be the same as at 60 cps? If its voltage regulation at rated load and 60 cps is 18 per cent, what will be the value of the voltage regulation at its rated load for 50-cps operation? The effect of armature-resistance voltage drop on regulation may be neglected.

6-24. A 300-hp 0.8-power-factor 2,300-volt, 76 amp, 60-cps, 3-phase synchronous motor has a direct-connected exciter to supply its field current. For the purposes of this problem, the efficiency of the exciter may be assumed constant at a value of 80 per cent. The synchronous motor is run at no load from a 2,300-volt 60-cps circuit, with its field current supplied by its exciter, and the following readings taken:

Armature voltage between terminals = 2,300 volts
Armature current = 76.0 amp per terminal
Three-phase power input = 27.5 kw
Field current = 30.0 amp
Voltage applied to field from armature terminals of the exciter = 300 volts

When the synchronous motor is loaded so that its input is 76 amp at 0.80 power factor and 2,300 volts between terminals, its field current is found to be 25 amp. Under these conditions, what is the efficiency of the synchronous motor exclusive to the losses in its exciter? What is the useful mechanical power output in horsepower?

6-25. A laboratory alternator is rated at 45 kva, 230 volts, 60 cps, 1,200 rpm, 3 phases and 0.80 power factor lagging and has the following open-circuit characteristic:

Line-to-line voltage, volts	0	180	220	260	300	340
Field current, amp	0	6.0	7.7	10.0	13.6	20.8

The stator effective resistance is 0.027 ohm per phase, and the synchronous reactance, adjusted for the approximate degree of saturation present under rated conditions, is 0.60 ohm per phase (both values for a Y connection).

a. What field current is required for operation under rated conditions?
b. To what value will the terminal voltage change if the load is thrown off while the field current remains constant at the value in part a?

6-26. A 50-hp 230-volt 60-cps, 1,200-rpm 3-phase 80-percent-power-factor synchronous motor has the open-circuit characteristic given in Problem 6-25. The synchronous reactance approximately adjusted for saturation can be taken as 0.60 ohm per phase (Y-connected). Rotational losses are 1,750 watts. Stator resistance is small compared with synchronous reactance, so that its effects can be ignored for present purposes.

a. What field current is required for operation under nameplate conditions?

b. At what power factor will the motor operate if the shaft output is halved while the field current remains constant at the value in part a? Consider the rotational losses to remain constant.

The stator effective resistance of the motor is 0.027 ohm per phase (Y-connected) and the field resistance is 5.0 ohms. Find the motor efficiency under (c) rated conditions and (d) half load.

6-27. Consider a salient pole synchronous motor with reactances X_d and X_q per unit. Assume the armature resistance is negligible. Derive the general equations for calculating the "V" curves. Write a computer program for this calculation. Calculate and plot the "V" curves for a machine where $X_d = 0.8$, $X_q = 0.6$ for connected shaft power of $P = 0, 0.5, 1.0, 1.25$ per unit.

6-28. Assume a cylindrical rotor synchronous generator connected to an infinitely large system and that the generator is supplying real power into the large system. Discuss, using phasor diagrams, the relationship between reactive power flow and excitation voltage.

6-29. An idealized 2-pole 3-phase synchronous machine is being driven at synchronous speed so that the angle from the axis of phase a to the d axis is $(\omega t + \alpha)$.

a. The armature terminals are open-circuited, and the field is excited by a step-function voltage V_f. Find the expressions for the instantaneous voltages of the armature phases and their d and q components.

b. Repeat a) if the transformer voltages, $\frac{d\lambda_d}{dt}$ and $\frac{d\lambda_q}{dt}$, are neglected.

c. Now consider that the field circuit is closed and unexcited ($v_f = 0$), the initial field and armature currents are zero, and $v_d = 0$. Neglect all resistances. At $t = 0$, a step-function voltage $v_q(t)$ of magnitude V_q is applied to the armature terminals. Find expressions for the armature currents i_d and i_q and the field current i_f. Introduce the transient inductance L'_d where appropriate.

6-30. A 3-phase turbine-generator is rated 13.8 kv (line to line), 110,000 kva. Its constants, with reactances expressed in per unit on the machine rating as a base, are

$$x_d = 1.10 \quad x'_d = 0.20 \quad T'_d = 1.0 \text{ sec}$$

It is operating unloaded at a terminal voltage of 1.00 per unit when a 3-phase short circuit occurs at its terminals. Ignore the second harmonic, and, except in part f, the d-c component in the short-circuit current. Express numerical answers both in per unit and in amperes.

a. What is the rms steady-state short-circuit current?

b. Write the numerical equation for the instantaneous phase a current as a function of time. Consider the fault to occur when $\theta_o = 90^o$. Because of the neglect of the d-c component, this is the symmetrical short-circuit current.

c. Write the numerical equation for the envelope of the short-circuit current wave as a function of time.

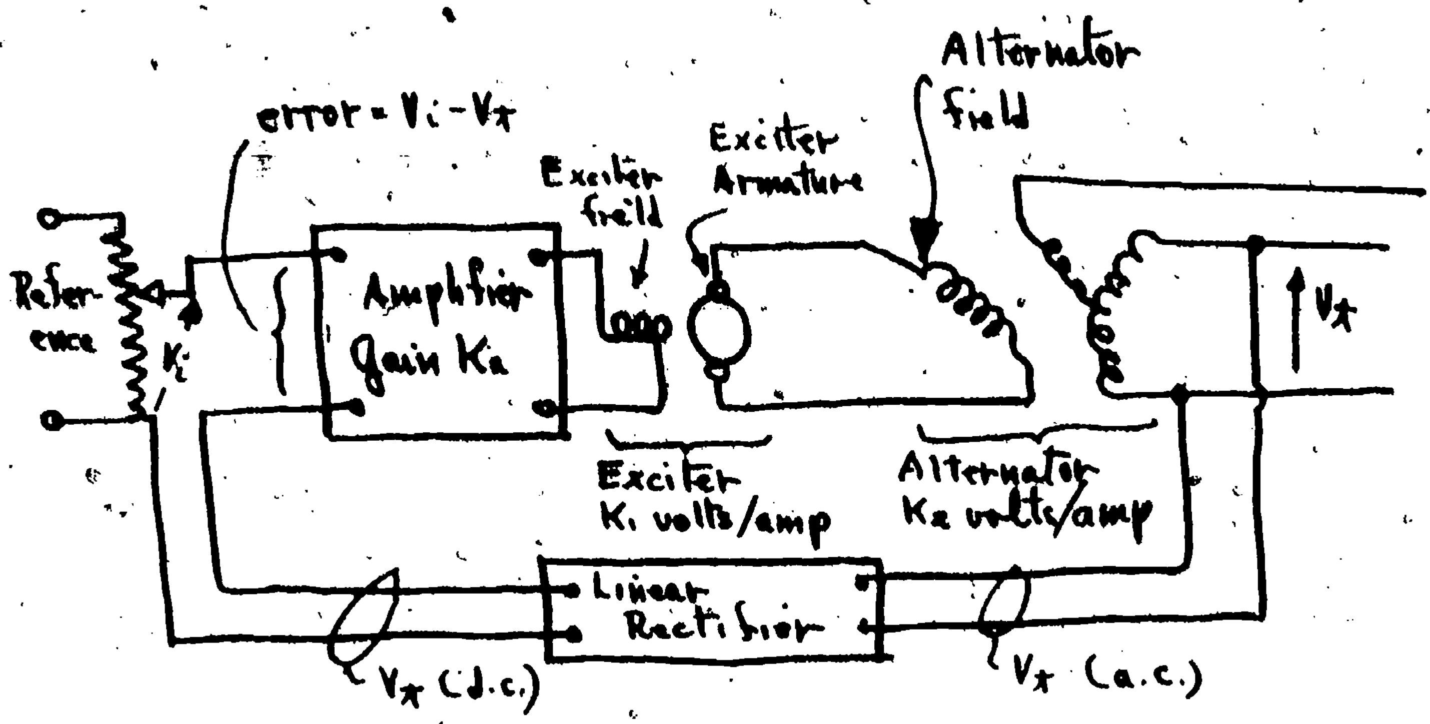
d. Using the results of part c, write the numerical equation showing how the rms value of short-circuit current varies with time.

e. What value is given by the expression in(d) at $t = 0$? This is known as the initial symmetrical rms short-circuit current.

f. In part b, suppose the fault occurs when the magnitude of θ_o is other than 90^o. The value of i_a at $t = 0$ would then be nonzero. But since the phase a winding is a resistance-inductance circuit, the complete phase a current cannot change instantaneously from zero. Hence a d-c component must be present in i_a to reconcile the situation. This component dies away rapidly. Give the maximum possible initial magnitude of the d-c component.

6-31. A proposed scheme for regulating the output voltage of a 5000-kva alternator is shown below. In addition to the alternator, the system includes a d-c exciter whose field excitation is obtained from the output of an amplifier. The input to the amplifier is the difference between the set value of voltage V_i and the actual output voltage V_t. The resistance and inductance of the exciter-field circuit (including the output impedance of the amplifier) are r_1 and L_1. The generated voltage of the exciter is k_1 volts/amp in its field circuit. The resistance and inductance of the alternator-field circuit (including the exciter armature) are r_2 and L_2. The alternator voltage is k_2 volts/amp in its field. For simplicity, the linear rectifier required to change the alternating voltage V_t to a direct voltage for comparison with V_i may be assumed to produce 1.0 volt d-c output per volt a-c input.

To examine the stability of the system, consider the alternator and exciter are rotating at rated speed but that initially their fields are unexcited and $V_i = 0$. The alternator is unloaded.



a. Show that the response of the system to a suddenly impressed value of V_i is characterized by the differential equation

$$\tau_1\tau_2 \frac{d^2V_t}{dt^2} + (\tau_1 + \tau_2)\frac{dV_t}{dt} + (1 + K)V_t = KV_i$$

Identify the time constants τ_1 and τ_2 and the static sensitivity K in terms of the system constants.

b. For a particular alternator and its excitation system, $\tau_1 = 0.4$ sec, and $\tau_2 = 1.0$ sec.

The system is to regulate within 0.5 per cent; i.e., the steady-state error in V_t is not to exceed 0.5 per cent of the set value V_t. Determine the damping constant ζ, the undamped naturel angular-frequency ω_n, and sketch the curve of the response V_t/V_i as a function of time.

6-32. Derive an expression for the instantaneous electromagnetic torque as a function of time after the occurrence of a 3-phase short circuit at the terminals of an unloaded machine. Express the result in terms of the initial field current i_{f0} and machine constants such as L'_d and L_q.

6-33. Reciprocating loads such as compressors require a torque which fluctuates periodically about a steady average value. For a 2-cycle unit, the torque harmonics have frequencies in cycles per second which are multiples of the speed in revolutions per second. When, as is commonly the case, the compressors are driven by synchronous motors, the torque harmonics cause periodic fluctuation of the torque angle δ and may result in undesirably high pulsations of power and current to the motor. It is therefore essential that, for the significant harmonics, the electrodynamic response of the motor to be held to a minimum.

a. To investigate the response of the motor to torque harmonics, use a linearized analysis similar to that yielding equation VI-424. Let

$$\Delta T_L = \Delta T_o + \Delta T_1 \sin \omega t + \ldots \Delta T_n \sin \omega n t$$

b. Show the relationship between amplitude of oscillations of δ and the machine parameters.

c. Assume a motor rated as follows:

200 hp, 2300 volt, 3 phase 60 cps 28 pole

Wk^2 = 10500 lb-ft^2 (motor and load)
Synchronizing power = 11.0 kw/electrical radians
Damping torque = 1770 lb-ft/mechanical radian per sec.
connected to a compressor load. The fundamental frequency torque oscillation amplitude is 580 lb-ft and at an angular velocity of 27.0 radians per sec.

d. Find the maximum deviation of δ.

e. What is the pulsation of synchronous power flow.

If a flywheel of wk^2 = 18000 lb-ft is added to the shaft, how does it affect the maximum deviation of δ ?

6-34. The ideal conditions for synchronizing an alternator with an electric power system ar that the alternator voltage be the same as that of the system bus in magnitude, phase, and frequency. Departure from these conditions results in undesirable current and power surges accompanying electromechanical oscillation of the alternator rotor. As long as the oscillations are not too violent, they may be investigated by a linearized analysis.

Consider that a 2,500-kw 0.80-power-factor 25-cps 26-pole oil-engine-driven alternator is to be synchronized with a 25-cps system large enough to be considered an infinite bus. The Wk^2 of the alternator, engine, and associated flywheel is 750,000 lb-ft^2. The damping-power coefficient C_d is 3,600 watts per electrical degree per second, and the synchronizing-power coefficient C_s is 1.21×10^5 watts per electrical degree. Both C_d and C_s may be assumed to remain constant. In all cases below, the terminal voltage is adjusted to its correct magnitude. The engine governor is sufficiently insensitive so that it does not act during the synchronizing period.

a. Consider that the alternator is initially adjusted to the correct speed but that it is synchronized out of phase by 20 electrical degrees, with the alternator leading the bus. Obtain a numerical expression for the ensuing electromechanical oscillations. Also give the largest value of torque exerted on the rotor during the synchronizing period. Ignore losses, and express this torque as a percentage of that corresponding to the nameplate rating.

b. Repeat (a) with the alternator synchronized at the proper angle but with its speed initially adjusted 1.0 cps fast.

c. Repeat (a) with the alternator initially leading the bus by 20 electrical degrees and its speed initially adjusted 1.0 cps fast.

6-35. A 60-cps synchronous alternator is driven by a four-cycle diesel engine. Owing to a misfire of one cylinder, a torque pulsation is produced every two revolutions of the machine. The pertinent data follow:

Alternator: 6,600 volts, 1,500 kva, 0.8 pf, 28 poles, 257 rpm; angle δ at full load = 30°

Mechanical system: Weight 16,000 kg; radius of gyration 0.95 m

Would you anticipate troublesome resonance effects?

6-36. In an industrial plant, load in the form of an air compressor is to be added. The compressor drive motor will be 500 hp, 3 phase, 460 volts, 60 cps. The average load prior to installation is 3650 kw at 70 per cent power factor lag. Because of the rate structure applicable to the plant it is necessary to raise the power factor to 90 per cent lagging..

The motor drive can be either induction or synchronous and an economic evaluation of the use of induction, unity power factor or 0.8 power factor leading synchronous motors will be made. If the induction motor is used, static capacities will be necessary. It may be necessary to use them with the synchrnous motors.

Cost of induction motor including control	\$ 6,700
Cost of unity pf synchronous motor, including control and exciter	\$10,400
Cost of 80 per cent leading power factor synchronous motor, including control and exciter	\$10,800
Power factor of induction motor	89% lag
Full load efficiency of all motors	90%
Cost of 460 volt capacitors	\$13/kva

All prices are fob factory with freight allowed. Installation costs are 9% of the delivered price of the induction motor, 12% of the price of the synchronous motor and $860 per kva for the capacitors.

Considering that all equipment has the same useful life, which motor is the recommended buy?

CHAPTER VII - POLYPHASE ASYNCHRONOUS MACHINES

7-1. The stator of a 10 pole, 3 phase squirrel cage induction motor is connected to a balanced 60 hz supply. For each of the following conditions - 1) at the instant of starting, 2) when rotor mechanical speed is 75% of synchronous speed and 3) at a slip of 0.04 find the following speeds in rpm:

a) speed of the stator field with respect to a stationary reference
b) speed of the stator field with respect to the rotor
c) speed of the rotor field with respect to a stationary reference
d) speed of the rotor field with respect to the stator field
e) speed of the rotor field with respect to the rotor

7-2. A two phase induction motor runs at very nearly 1000 rpm at no load and at 950 rpm at full load when energized from a 50 hz supply.

a) how many poles has the motor?
b) what is the slip at full load?
c) what is the frequency of the rotor currents?
d) what is the speed of the rotor field with respect to the rotor?
e) repeat the above for a slip of 8%.

7-3. A frequency-changer set is to be designed for supplying variable-frequency power to induction motors driving the propellors on scale-model airplanes for wind-tunnel testing. The frequency changer is a wound-rotor induction machine driven by a d-c motor whose speed can be controlled. The 3-phase stator winding of the induction machine is excited from a 60 hz source, and variable-frequency 3-phase power is taken from its rotor winding. The set must meet the following specifications:

Output frequency range = 120 to 450 hz.
Maximum speed not to exceed 3,000 rpm
Maximum power output = 100 kw at 0.25 power factor and 450 hz

The power required by the induction-motor load drops off rapidly with decreasing frequency, so that the maximum-speed condition determines the sizes of the machines.

On the basis of negligible exciting current, losses, and voltage drops in the induction machine, find:

a. The minimum number of poles for the induction machine
b. The corresponding maximum and minimum speeds
c. The kva rating of the stator winding of the induction machine
d. The horsepower rating of the d-c machine

7-4. Electrical power is to be supplied to a 3-phase 25 hz system from a 3-phase 60 hz system through a motor-generator set consisting of two directly coupled synchronous machines.

a. What is the minimum number of poles which the motor may have?
b. What is the minimum number of poles which the generator may have?
c. At what speed in rpm will the set specified in (a) and (b) operate?
d. Can you suggest a more reasonable combination (from the standpoint of number of poles) which might be possible if a gear ratio between generator and motor is used?

7-5. In Europe, where 50 hz systems are universally used, the electrified railways use power at 16 2/3 hz, obtained from an induction type frequency converter. Discuss why this particular combination is advantageous.

7-6. An Electropult, (See Westinghouse Engr., September 1946, p. 161), based on the induction-motor principle may be used for launching heavily loaded airplanes from short runways. It consists of a launching car riding on a long track. The track is a developed squirrel-cage winding, and the launching car, which is 12 ft long, 31/2 ft wide, and only 5 1/2 in. high, has a developed 3-phase 8-pole winding. The center-line distance between adjacent poles is 12/8 = 1 1/2 ft. Power at 60 hz is fed to the car from arms extending through slots to rails below ground level. The car develops 10,000 hp and can launch an airplane in as little as 4 sec over a 340-ft run.

a. What is the synchronous speed in miles per hour?
b. Will the car reach this speed? Explain your answer.
c. To what slip frequency does a car speed of 75 mph correspond?
d. The resistance of the bars in the squirrel-cage track winding diminish from a maximum at the start of the runway to a minimum where the airplane leaves the runway. Explain the purpose and the effect of this construction.
e. As soon as the airplane is launched, direct current is applied to the 3-phase winding. Explain what the effect of this would be.

7-7. The stator of an unloaded 3-phase 6-pole wound-rotor induction motor is connected to a 60 hz source; the rotor is connected to a 25 hz source.

a. Is a starting torque produced?
b. At what speed will steady-state motor action result? (there are two possible answers).
c. What determines at which of the two speeds in (b) the motor will operate in a specific case?
d. Suppose now that the rotor supply frequency is varied over the range 0 to 25 hz. Sketch curves showing motor speed in rpm as a function of rotor frequency, interpreting zero frequency as direct current.
e. What changes are made in the foregoing answers if the motor is fully loaded instead of unloaded?

7-8. This problem is concerned with analysis of a 2-phase induction machine instead of the 3-phase machine. Synchronously rotating dq axes are still used. There are, however, 2 distributed windings a and b on the stator, 1 in each phase, with the magnetic axis of phase b 90° ahead of the phase a axis in the direction of rotation. There are also 2 corresponding windings A and B on the rotor, also with axes 90° apart. At any instant of time, the axis of a rotor phase is displaced by the angle θ_2 from the axis of the correspondingly lettered stator phase.

a. Write the phase-linkage equations corresponding to (VII-31) to (VII-36).

b. Show that the appropriate dq transformation of variables is typified by the following:

Stator:

$$i_{1d} = i_a \cos \omega t + i_b \sin \omega t$$

$$i_{1q} = - i_a \sin \omega t + i_b \cos \omega t$$

Rotor:

$$i_{2d} = i_A \cos \theta_s + i_B \sin \theta_s$$

$$i_{2q} = - i_A \sin \theta_s + i_B \cos \theta_s$$

Also, write the reverse equations for phase variables in terms of dq variables.

c. Show that the dq linkage relations are still given by (VII-44) through (VII-47).

d. Show that the component voltage relations of (VII-40) through (VII-43) are correct.

e. Show that the instantaneous power input to the 2-phase stator is

$$p_1 = v_{1d} i_{1d} + v_{1q} i_{1q}$$

f. Show that the motor torque is given by

$$T = \frac{\text{poles}}{2} (\lambda_{2q} i_{2d} - \lambda_{2d} i_{2q})$$

7-9. Equations (VII-40) through (VII-47) constitute the basic electrical relations in dq variables for an induction machine with no zero-sequence currents. On the basis of these equations, set up a block diagram to represent the electrical performance of the machine. The vaoltages v_{1d} and v_{1q} are signals to be received from the stator supply system. The slip angular velocity $p\theta_s = s\omega$ is either constant or a signel to be received from a representation of the mechanics of the machine and the equipment coupled to its shaft. Constraints are placed on the rotor component voltages and currents, not only by the machine, but also by the circuitry or equipment connected to the rotor terminals.

7-10. Modify the block diagram of Prob.. 7-9 so that it represents the squirrel cage configuration (with a short-circuited rotor winding).

7-11. A 100-hp 3-phase Y-connected 440-volt 60 hz 6-pole squirrel-cage induction motor has the following equivalent-circuit constants in ohms per phase referred to the stator:

$$r_1 = 0.085 \qquad r_2 = 0.067$$

$$x_1 = 0.196 \qquad x_2 = 0.161$$

$$x_{12} = 6.65$$

No-load rotational loss = 2.7 kw. Stray load loss = 0.5 kw. The rotational and stray load losses may be considered constant.

a. Compute the horsepower output, stator current power factor, and efficiency at rated voltage and frequency for a slip of 3.00 per cent.
b. Compute the starting current and the internal starting torque in pound-feet at rated voltage and frequency

7-12. A 50-hp 60 hz, 6-pole induction motor runs at a slip of 3.0 per cent at full load. Rotational and stray load losses at full load are 4.0 per cent of the output power. Compute:

a. The rotor copper loss at full load
b. The electromagnetic torque at full load, in newton-meters
c. The power delivered by the stator to the air gap at full load

7-13. A 100-hp 230-volt 3-phase Y-connected 60 hz 4-pole squirrel-cage induction motor develops full-load internal torque at a slip of 0.04 when operated at rated voltage and frequency. For the purposes of this problem rotational and core losses can be neglected. Impedance data on the motor are as follows:

Stator resistance r_1 = 0.036 ohm per phase
Leakage reactance $x_1 = x_2$ = 0.047 ohm per phase
Magnetizing reactance x_{12} = 15.5 ohms per phase

Determine the maximum internal torque at rated voltage and frequency, the slip at maximum torque, and the internal starting torque at rated voltage and frequency. Express the torques in newton-meters.

7-14. A 3-phase induction motor, at rated voltage and frequency, has a starting torque of 180 per cent and a maximum torque of 250 per cent of full-load torque. Neglect stator resistance and rotational losses, and assume constant rotor resistance. Determine:

a. The slip at full load
b. The slip at maximum torque
c. The rotor current at starting, in per unit of full load rotor current

7-15. A 100-hp 6-pole 60 hz high-starting-torque 3-phase induction motor produces rated horsepower at rated voltage and frequency with 10 per cent slip. This motor can develop 300 per cent pull-out torque based on rated full-load torque.

The motor is used to drive a load requiring a torque directly proportional to speed; at 2,000 rpm, the load torque is 700 lb-ft. Consider the torque-speed curve of the motor a straight line between 0 and 15 per cent slip.

a. Sketch the motor and load torque-speed characteristics to approximate scale.
b. At what speed will the motor drive this load? Applied voltage is rated value.
c. While the motor is driving the same load, the voltage applied to the motor is raised 10 per cent. Show whether or not the motor will be loaded beyond its horsepower rating.
d. The rotor is replaced by one having twice the resistance per phase but identical in all other respects with the old one. What will be the motor speed with rated voltage applied to the motor?

7-16. When operated at rated voltage and frequency, a 3-phase squirrel-cage induction motor (of the design classification known as a high-slip motor) delivers full load at a slip of 10 per cent and develops a maximum torque of 300 per cent of full-load torque at a slip of 60 per cent. Neglect core and rotational losses, and assume that the resistances and inductances of the motor are constant.

Determine the torque and rotor current at starting with rated voltage and frequency. Express the torque and rotor current in per unit based on their full-load values.

7-17. A polyphase induction motor has negligible rotor rotational losses and is driving a pure-inertia load. The moment of inertia of the rotor plus load is J mks units.

a. Obtain an expression for the rotor energy loss during starting. Express the result in terms of J and the synchronous angular velocity ω_s.
b. Obtain an expression for the rotor energy loss associated with reversal from full speed forward by reversing the phase sequence of the voltage supply (a process known as plugging). Express the results in terms of J and ω_s.
c. State and discuss the degree of dependence of the results in (a) and (b) on the current-limiting scheme which may be used during starting and reversal. (Neglect rotational and stray load losses andconsider that acceleration is slow enough to permit application of steady state theory).

7-18. A 4-pole 440-volt 400-hp 3-phase 60 hz Y-connected wound-rotor induction motor has the following constants in ohms per phase referred to the stator: $x_1 = x_2 = 0.055$, $x = 2.23$, $r_1 = 0.0054$, $r_2 = 0.0071$. The motor is supplied at normal terminal voltage through a series reactance of 0.03 ohm per phase representing a step-down transformer bank. It is fully loaded, the slip rings are short-circuited, and the efficiency and power factor are 90.5 and 90.0 per cent, respectively.

A 3-phase short circuit occurs at the high-tension terminals of the transformer bank. Determine the initial symmetrical short-circuit current in the motor, and show how it is decremented.

7-19. The following test data apply to 50-hp, 2,300-volt 60-hz 3-phase squirrel-cage induction motor:

No-load Test at Rated Voltage and Frequency:

Line current = 4.1 amp
3-phase power = 1,550 watts

Blocked-rotor Test at 15 hz

Line voltage = 268 volts
Line current = 25.0 amp
3-phase power = 9,600 watts
Stator resistance between line terminals = 5.80 ohms
Stray load loss = 420 watts

Compute the stator current and power factor, horsepower output, and efficiency when this motor is operating at rated voltage and frequency with a slip of 0.04.

7-20. A 50000 hp, 50 hz, 22 pole wound-rotor induction motor is used to drive a wind-tunnel fan. The power required to drive the fan varies as the cube of its speed and is 50000 hp at 297 rpm.

The speed must be adjustable over a range of 297 to 37.5 rpm. If rotor-resistance speed control is used, plot curves of the following variables as functions of the speed in rpm:

a. Fan power, in kilowatts
b. Power input to the motor, in kilowatts. Neglect stator copper, rotational, and stray load losses
c. Total rotor-circuit copper loss, in kilowatts.

7-21. A 220-volt 3-phase 4-pole 60 hz induction motor develops a maximum torque of 225 per cent of the full-load torque at a slip of 0.15 when operated at rated voltage and frequency. The actual rotor resistance (not the referred value) is 0.03 ohm per phase. The stator resistance and rotational losses can be neglected.

a. What external resistance in actual ohms should be inserted in each phase of the rotor winding in order to give maximum torque at starting?

b. What is the slip at full load without the external rotor resistance? Would this slip be larger, the same, or smaller if the stator resistance were considered?
c. What are the slip and torque in per cent when the motor current has its full-load value but resistances of 0.07 ohm are inserted in each phase of the rotor circuit?
d. What would be the maximum torque in per cent of the full-load torque if the motor were connected to a 200-volt 50-hz source?

7-22. A 220-volt 3-phase 4-pole 60-hz, 100-hp squirrel-cage induction motor takes a blocked-rotor current of 200 per cent and develops an internal starting torque of 16 per cent for an applied voltage of 30 per cent. A starting compensator is to be purchased for this motor. The starting compensator may be regarded as an ideal 3-phase step-down transformer connected between the supply line and the motor. Determine the per cent starting torque if the starting compensator limits the starting current in the supply line to 150 per cent of the motor full-load current. The supply-line voltage is 220 volts.

7-23. An induction motor has a full voltage locked rotor current of 7.0 per unit. It is desired to limit the maximum current, on starting, from the supply to 3 per cent. Two schemes are being considered - auto transformers starting or series reactance inserted for current limiting.

a. What is the transformer turns ratio required?
b. What is the per unit reactance required?
c. What is the ratio of starting torques developed in the two methods?

7-24. A 230-volt Y-connected 3-phase 6-pole 60-hz wound-rotor induction motor has a stator-plus-rotor leakage reactance of 0.50 ohm per phase referred to the stator, a rotor-plus-load moment of inertia of 1.0 kg-m^2, negligible losses (except for rotor copper loss), and negligible exciting current. It is connected to a balanced 230volt source and drives a pure-inertia load. Across-the-line starting is used, and the rotor-circuit resistance is to be adjusted so that the motor brings its load from rest to one-half synchronous speed in the shortest possible time.

Determine the value of the rotor resistance referred to the stator and the minimum time to reach one-half of synchronous speed.